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ON CONTRA Λ_I^s-CONTINUOUS FUNCTIONS AND THEIR APPLICATIONS

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Abstract. In this work, we introduce and study the classes of contra Λ_I^s-continuous, contra quasi-Λ_I^s-continuous and contra Λ_I^s-irresolute functions in a topological space endowed with an ideal. We investigate the relationships among these functions and their respective characterizations. Also, we analyze the behavior of certain topological notions under direct and inverse images of these new classes of functions.

Keywords: ideal; semi-I-open set; Λ_I^s-closed set; contra Λ_I^s-irresolute function.

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1. Introduction

In 1933, Kuratowski [5] introduced the notion of local function of a set as a generalization of the closure of a set. In this sense, given a topological space (X,τ) and an ideal I on X, the local function of a subset A of X with respect to I and τ, is the set $A^\star = \{x \in X : U \cap A \notin$

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I for each $U \in \tau$ such that $x \in U\}$. In 1990, Jankovic and Hamlett [4], used this generalization in order to define the Kuratowski operator $Cl^\star$, which induces a topology $\tau^\star$ finer than τ. Using the operator $Cl^\star$, Hatir and Noiri [3], in 2002, defined the concept of semi-I-open set and used it to establish some decompositions of generalized continuity. Later, in 2013, Sanabria et al. [6] used the class of semi-I-open sets to define and study the notions of Λ_I^s- sets and Λ_I^s -closed sets, in particular, through these notions they characterized the semi-I-T_1-spaces and the semi-I-$T_{1/2}$-spaces. Quite recently Sanabria et al. [7] used the class of Λ_I^s-closed sets to define and characterize new variants of continuity called Λ_I^s-continuous, quasi-Λ_I^s-continuous and Λ_I-irresolute functions. On the other hand, in 1996, Dontchev [1] introduced the notion of contra-continuous function in topological spaces and, established interesting results that related contra-continuity with compact spaces, S-closed spaces and strongly S-closed spaces. In this work, we introduce and characterize new variants of contra-continuous functions defined in terms of Λ_I^s-closed sets, in contrast with the variants of continuity due to Sanabria et al. [7]. Specifically, are studied the concepts of contra Λ_I^s-continuous, contra Λ_I^s-irresolute and contra quasi-Λ_I^s-continuous functions. The preservation of certain modifications of connectedness, compactness and separations are investigated, through direct and inverse images of these functions.

2. PRELIMINARIES

Throughout this paper, $Int(A)$ and $Cl(A)$ denote the interior and the closure of a subset A of a topological space (X,τ), respectively. Also $P(X)$ denote the power set of X. An *ideal* I on X is a nonempty subfamily of $P(X)$ which satisfies the following conditions:

(1) If $A \subset B$ and $B \in I$, then $A \in I$.

(2) If $A \in I$ and $B \in I$, then $A \cup B \in I$.

Some well-known types of ideals on a topological space (X,τ) are the family of all nowhere dense subsets of X, the family of all closed and discrete subsets of X and the family of all meager subsets of X.

In the sequel, (X,τ,I) (or simply X) denote a topological space (X,τ) with an ideal I on X, which is simply called an ideal space. Given an ideal space (X,τ,I) and a subset A of X, the *local function* of A with respect to I and τ is defined as $A^\star(I,\tau) = \{x \in X : U \cap A \notin$

I for each $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau : x \in U\}$. When there is no chance for confusion, we will simply write $A^\star$ instead of $A^\star(I,\tau)$. It is a well known fact that $Cl^\star(A) = A \cup A^\star$ is a Kuratowski closure operator and hence, $\tau^\star(I,\tau) = \{U \subset X : Cl^\star(X-U) = X-U\}$ is a topology on X, which is finer than τ. If there is no chance to confusion, we will write $\tau^\star$ instead $\tau^\star(I,\tau)$. The members of $\tau^\star$ are called *$\tau^\star$-open* sets and their complements are called *$\tau^\star$-closed* sets. It is easy to see that a subset A of X is $\tau^\star$-closed if and only if $A^\star \subset A$. According to [3], $A \subset X$ is *semi-I-open* if $A \subset Cl^\star(Int(A))$. The family of all semi-I-open sets of X is denoted by SIO(X,τ) and the complement of a semi-I-open set is called *semi-I-closed* set. As in [6], we say that $A \subset X$ is a *Λ_I^s-set* if $A = \Lambda_I^s(A)$, where $\Lambda_I^s(A)$ is the intersection of all semi-I-open subsets of X containing A. According also to [6], we say that $A \subset X$ is *Λ_I^s-closed* if $A = L \cap F$, where L is a Λ_I^s-set and F is a $\tau^\star$-closed set. In [6], the following implications were established:

$$\begin{array}{ccccc} \text{open} & \Longrightarrow & \text{semi-}I\text{-open} & \Longrightarrow & \Lambda_I^s\text{-set} \\ & & & & \Downarrow \\ \text{closed} & \Longrightarrow & \tau^\star\text{-closed} & \Longrightarrow & \Lambda_I^s\text{-closed} \end{array}$$

The complement of a Λ_I^s-closed set is called *Λ_I^s-open* set. According to the previous implications, we have the following results.

Lemma 2.1. [7, Lemma 2.2] *Every $\tau^\star$-open set is Λ_I^s-open.*

Lemma 2.2. [7, Lemma 2.3] *Let $\{A_\alpha : \alpha \in \Delta\}$ be a collection of subsets of a space (X,τ,I). If A_α is a Λ_I^s-open set for each $\alpha \in \Delta$, then $\bigcup\{A_\alpha : \alpha \in \Delta\}$ is a Λ_I^s-open set.*

3. Contra Continuity in Terms Of Λ_I^s-Closed Sets

In this section, we introduce and study certain types of contra continuity in terms of Λ_I^s-closed sets. We establish some relationships and characterizations for these new classes of functions. Next we consider a function f defined from an ideal space (X,τ,I) to an ideal space (Y,σ,J). First, we present the definitions and characterizations of some variants of continuity due to Sanabria et al. [7].

Definition 3.1. [7] A function $f : (X,\tau,I) \longrightarrow (Y,\sigma,J)$ is called:

(1) *Λ_I^s-continuous*, if $f^{-1}(V)$ is a Λ_I^s-open subset of X for each open subset V of Y.

(2) *Quasi-Λ_I^s-continuous*, if $f^{-1}(V)$ is a Λ_I^s-open subset of X for each $\sigma^\star$-open set V of Y.

(3) *Λ_I^s-irresolute*, if $f^{-1}(V)$ is a Λ_I^s-open set of X for each Λ_I^s-open subset V of Y.

Theorem 3.1. [7] *For a function $f:(X,\tau,I)\to(Y,\sigma,J)$, the following statements are equivalent:*

(1) *f is Λ_I^s-continuous (resp. quasi-Λ_I^s-continuous, Λ_I^s-irresolute).*

(2) *$f^{-1}(B)$ is a Λ_I^s-closed subset of X for each closed (resp. $\sigma^\star$-closed, Λ_J^s-closed) subset B of Y.*

(3) *For each $x\in X$ and each open (resp. $\sigma^\star$-open, Λ_J^s-open) subset V of Y containing $f(x)$ there exists a Λ_I^s-open subset U of X containing x such that $f(U)\subset V$.*

Now, we introduce a new variants of contra-continuity and study the relationships among these classes of functions and also obtain properties and characterizations of them.

Definition 3.2. A function $f:(X,\tau,I)\longrightarrow(Y,\sigma,J)$ is called:

(1) *Contra Λ_I^s-continuous*, if $f^{-1}(V)$ is a Λ_I^s-closed subset of X for each open set V of Y.

(2) *Contra quasi-Λ_I^s-continuous*, if $f^{-1}(V)$ is a Λ_I^s-closed subset of X for $\sigma^\star$-open subset V of Y.

(3) *Contra Λ_I^s-irresolute*, if $f^{-1}(V)$ is a Λ_I^s-closed subset of X for each Λ_J^s-open subset V of Y.

Theorem 3.2. *Let $f:(X,\tau,I)\longrightarrow(Y,\sigma,J)$ be a function. The following statements hold:*

(1) *If f is contra Λ_I^s-irresolute, then it is contra quasi-Λ_I^s-continuous.*

(2) *If f is contra quasi-Λ_I^s-continuous, then it is contra Λ_I^s-continuous.*

(3) *If f is contra Λ_I^s-irresolute, then it is contra Λ_I^s-continuous.*

Proof. (1) Let V be any $\sigma^\star$-open subset of Y. By Lemma 2.1, we have V is a Λ_J^s-open subset of Y and since f is contra Λ_I^s-irresolute, it follows that $f^{-1}(V)$ is a Λ_I^s-closed subset of X. Therefore, f is contra quasi-Λ_I^s-continuous.
The proof of (2) is similar to case (1) and, (3) is an immediate consequence of (1) and (2). □

In the following two examples, we show that the converses of (1) and (2) in Theorem 3.2, in general are not true.

Example 3.1. Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}, \{a,b\}\}$, $\sigma = \{\emptyset, \{c\}, X\}$, $I = \{\emptyset, \{b\}, \{a,b\}\}$ and $J = \{\emptyset, \{a\}\}$. Then, the familly of all $\sigma^\star$-open subsets of X is $\{\emptyset, X, \{c\}, \{b,c\}\}$, the family of all Λ^s_J-open subsets of X is $\{\emptyset, X, \{c\}, \{b,c\}, \{a,c\}\}$, the family of all Λ^s_I-closed subsets of X is $\{\emptyset, X, \{a\}, \{a,b\}, \{c\}, \{a,c\}\}$ and, we have the identity function $f : (X, \tau, I) \to (X, \sigma, J)$ is contra quasi-Λ^s_I-continuous, but is not contra Λ^s_I-irresolute, because $f^{-1}(\{b,c\})$ is not a Λ^s_I-closed subset of X.

Example 3.2. Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a,c\}\}$, $\sigma = \{\emptyset, X, \{a,c\}, \{b\}\}$, $I = \{\emptyset, \{b\}\}$ and $J = \{\emptyset, \{a\}\}$. Then, the family of all $\sigma^\star$-open subsets of X is $\{\emptyset, X, \{b\}, \{c\}, \{a,c\}, \{b,c\}\}$, the family of all Λ^s_I-closed subsets of X is $\{\emptyset, X, \{a,c\}, \{b\}\}$ and, we have the identity function $f : (X, \tau, I) \to (Y, \sigma, J)$ is contra Λ^s_I-continuous, but is not contra quasi-Λ^s_I-continuous, because $f^{-1}(\{c\})$ and $f^{-1}(\{b,c\})$ are not Λ^s_I-closed subsets of X.

From Theorem 3.2, Example 3.1 and Example 3.2, we have the following diagram, where the implications are not reversible:

$$\text{Contra } \Lambda^s_I\text{-irresolute} \Longrightarrow \text{Contra quasi-}\Lambda^s_I\text{-continuous} \Longrightarrow \text{Contra } \Lambda^s_I\text{-continuous}$$

Theorem 3.3. *For a function $f : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are equivalent:*

(1) *f is contra Λ^s_I-continuous.*

(2) *$f^{-1}(B)$ is a Λ^s_I-open subset of X for each closed subset B of Y.*

(3) *For each $x \in X$ and each closed subset B of Y containing $f(x)$, there exists a Λ^s_I-open subset U of X such that $x \in U$ and $f(U) \subset B$.*

Proof. (1) $\Rightarrow$ (2) Let B be any closed subset of Y. Then $V = Y - B$ is an open subset of Y and, as f is contra Λ^s_I-continuous, $f^{-1}(V)$ is a Λ^s_I-closed subset of X. Since $f^{-1}(V) = f^{-1}(Y - B) = f^{-1}(Y) - f^{-1}(B) = X - f^{-1}(B)$, we conclude $f^{-1}(B)$ is a Λ^s_I-open subset of X.

(2) $\Rightarrow$ (1) Let V any open subset of Y. Then $B = Y - V$ is a closed subset of Y and, by hypothesis, $f^{-1}(B)$ is a Λ^s_I-open subset of X. Since $f^{-1}(B) = f^{-1}(Y - V) = f^{-1}(Y) - f^{-1}(V) = X - f^{-1}(V)$, we obtain that $f^{-1}(V)$ is a Λ^s_I-closed subset of X and hence, f is contra Λ^s_I-continuous.

(1) $\Rightarrow$ (3) Let $x \in X$ and B be any closed subset of Y such that $f(x) \in B$. Then $x \in f^{-1}(B)$ and, because f is contra Λ^s_I-continuous, $f^{-1}(B)$ is a Λ^s_I-open subset of X. If $U = f^{-1}(B)$, then U is

a Λ_I^s-open subset of X such that $x \in U$ and $f(U) = f(f^{-1}(B)) \subset B$.

(3) $\Rightarrow$ (1) Let B any closed subset of Y and let $x \in f^{-1}(B)$. Then $f(x) \in B$ and, by hypothesis, there exists a Λ_I^s-open subset U_x of X such that $x \in U_x$ and $f(U_x) \subset B$. Thus, $x \in U_x \subset f^{-1}(f(U)) \subset f^{-1}(B)$ and hence, $f^{-1}(B) = \bigcup\{U_x : x \in f^{-1}(B)\}$. By Lemma 2.2, we obtain that $f^{-1}(B)$ is a Λ_I^s-open subset of X. □

Theorem 3.4. *For a function $f : (X,\tau,I) \to (Y,\sigma,J)$, the following statements are equivalent:*

(1) *f is contra quasi-Λ_I^s-continuous.*

(2) *$f^{-1}(B)$ is a Λ_I^s-open subset of X for each $\sigma^\star$-closed subset B of Y.*

(3) *For each $x \in X$ and each $\sigma^\star$-closed subset B of Y containing $f(x)$, there exists a Λ_I^s-open subset U of X such that $x \in U$ and $f(U) \subset B$.*

Proof. The proof is similar to that of Theorem 3.3. □

Theorem 3.5. *For a function $f : (X,\tau,I) \to (Y,\sigma,J)$, the following statements are equivalent:*

(1) *f is contra Λ_I^s-irresolute.*

(2) *$f^{-1}(B)$ is a Λ_I^s-open subset of X for each Λ_J^s-closed subset B of Y.*

(3) *For each $x \in X$ and each Λ_J^s-closed subset B of Y containing $f(x)$, there exists a Λ_I^s-open subset U of X such that $x \in U$ and $f(U) \subset B$.*

Proof. The proof is similar to that of Theorem 3.3. □

Theorem 3.6. *Let $f : (X,\tau,I) \to (Y,\sigma,J)$ and $g : (Y,\sigma,J) \to (Z,\theta,K)$ be two functions, where I, J and K are ideals on X, Y and Z, respectively. Then:*

(1) *$g \circ f$ is contra Λ_I^s-irresolute, if f is Λ_I^s-irresolute and g is contra Λ_J^s-irresolute.*

(2) *$g \circ f$ is contra Λ_I^s-irresolute, if f is contra Λ_I^s-irresolute and g is Λ_J^s-irresolute.*

(3) *$g \circ f$ is Λ_I^s-irresolute, if f is contra Λ_I^s-irresolute and g is contra Λ_J^s-irresolute.*

(4) *$g \circ f$ is contra Λ_I^s-continuous, if f is contra Λ_I^s-continuous and g is continuous.*

(5) *$g \circ f$ is contra Λ_I^s-continuous, if f is contra Λ_I^s-irresolute and g is Λ_J^s-continuous.*

(6) *$g \circ f$ is contra Λ_I^s-continuous, if f is Λ_I^s-irresolute and g is contra Λ_J^s-continuous.*

(7) *$g \circ f$ is contra Λ_I^s-continuous, if f is Λ_I^s-continuous and g is contra continuous.*

(8) *$g \circ f$ is Λ_I^s-continuous, if f is contra Λ_I^s-continuous and g is contra continuous.*

(9) *$g\circ f$ is Λ_I^s-continuous, if f is contra Λ_I^s-irresolute and g is contra Λ_J^s-continuous.*

(10) *$g\circ f$ is contra quasi-Λ_I^s-continuous, if f is Λ_I^s-irresolute and g is contra quasi-Λ_J^s-continuous.*

(11) *$g\circ f$ is contra quasi-Λ_I^s-continuous, if f is contra Λ_I^s-irresolute and g is quasi Λ_J^s-continuous.*

(12) *$g\circ f$ is quasi Λ_I^s-continuous, if f is contra Λ_I^s-irresolute and g is contra quasi-Λ_J^s-continuous.*

Proof. (1) Let V be a Λ_K^s-open subset of Z. Since g is contra Λ_J^s-irresolute, $g^{-1}(V)$ is a Λ_J^s-closed subset of Y and, as f is Λ_I^s-irresolute, then by Theorem 3.1, we have $f^{-1}(g^{-1}(V))$ is a Λ_I^s-closed subset of X. Because $(g\circ f)^{-1}(V)=(f^{-1}\circ g^{-1})(V)=(f^{-1}(g^{-1}(V))$, it follows that $(g\circ f)^{-1}(V)$ is a Λ_I^s-closed subset of X. This shows that $g\circ f$ is contra Λ_I^s-irresolute.

The proofs of (2)-(12) are similar to that of (1). □

Theorem 3.7. *Let $f:(X,\tau,I)\to(Y,\sigma)$ be a function and let μ be the product topology induced by τ and σ on $X\times Y$. Let $g:(X,\tau,I)\to(X\times Y,\mu)$ be the graph function of f defined by $g(x)=(x,f(x))$ for each $x\in X$. If g is contra Λ_I^s-continuous, then f is contra Λ_I^s-continuous.*

Proof. Let V be any open subset of Y. Then $X\times V$ is a open subset of $X\times Y$ and, as g is contra Λ_I^s-continuous, we have $f^{-1}(V)=g^{-1}(X\times V)$ is a Λ_J^s-closed subset of X. Therefore, f is contra Λ_I^s-continuous. □

4. Study Of Direct and Inverse Images by Contra Λ_I^s-Continuous Functions

In this section, we study the behavior of some modifications of classical topological notions under direct and inverse images of the new variants of contra continuity introduced in the previous section. Recall that an ideal space (X,τ,I) is said to be Λ_I^s-*connected* (resp. $\tau^\star$-*connected*), if X cannot be written as a disjoint union of two nonempty Λ_I^s-open (resp. $\tau^\star$-open) sets, see [7].

Theorem 4.1. *If $f:(X,\tau,I)\to(Y,\sigma)$ is a surjective contra Λ_I^s-continuous function and (X,τ,I) is a Λ_I^s-connected space having more than one element, then (Y,σ) is a not discrete space.*

Proof. Suppose that (Y,σ) is a discrete space and let A be any nonempty proper subset of Y. Then, A is a clopen subset of Y and as f is contra Λ_I^s-continuous, $f^{-1}(A)$ is a Λ_I^s-open and Λ_I^s-closed subset of X. Since (X,τ,I) is a Λ_I^s-connected space, by [7, Theorem 4.5], the only subsets of X which are both Λ_I^s-open and Λ_I^s-closed are $\emptyset$ and X. Thus, $f^{-1}(A)=\emptyset$ or $f^{-1}(A)=X$. If $f^{-1}(A)=\emptyset$, then this contradicts the hypothesis that $A\neq\emptyset$ and f is surjective. If $f^{-1}(A)=X$, since f is surjective and A is a proper subset of Y, there exist $y_0\in Y-A$ and $x_0\in X$ such that $y_0=f(x_0)$, but then $y_0\in A$ and $y_0\notin A$. Again we obtain a contradiction. Therefore, (Y,σ) is not a discrete space. □

Theorem 4.2. *Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be a surjective function. The following statements hold:*

(1) *If f is contra Λ_I^s-irresolute and (X,τ,I) is a Λ_I^s-connected space, then (Y,σ,J) is a Λ_J^s-connected space.*

(2) *If f is contra quasi-Λ_I^s-continuous and (X,τ,I) is a Λ_I^s-connected space, then (Y,σ,J) is a $\sigma^\star$-connected space.*

(3) *If f is contra Λ_I^s-continuous and (X,τ,I) is a Λ_I^s-connected, then (Y,σ) is a connected space.*

Proof. (1) Suppose that (X,τ,I) is a Λ_I^s-connected space and $f:(X,\tau,I)\to(Y,\sigma,J)$ is a surjective contra Λ_I^s-irresolute function. If (Y,σ,J) is not Λ_J^s-connected, there exist two nonempty Λ_J^s-open subsets A and B of Y such that $A\cap B=\emptyset$ and $Y=A\cup B$, which implies that $B=Y-A$ and $A=Y-B$ are nonempty Λ_J^s-closed subsets of Y and, because f is contra Λ_I^s-irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint Λ_I^s-open subsets of X such that $X=f^{-1}(A)\cup f^{-1}(B)$. This contradicts the fact that (X,τ,I) is Λ_I^s-connected. Therefore, (Y,σ,J) is Λ_J^s-connected.

The proofs of (2) and (3) are similar to that of (1). □

Theorem 4.3. *Let (Y,σ) be any T_0-space. If each contra Λ_I^s-continuous function $f:(X,\tau,I)\to(Y,\sigma)$ is constant, then (X,τ,I) is a Λ_I^s-connected space.*

Proof. Suppose that (X,τ,I) does not is a Λ_I^s-connected space and each contra Λ_I^s-continuous function $f:(X,\tau,I)\to(Y,\sigma)$ is constant. By [7, Theorem 4.5], there exists a proper nonempty subset A of X which is both Λ_I^s-open and Λ_I^s-closed. Let $Y=\{a,b\}$ endowed with the topology $\sigma=\{Y,\emptyset,\{a\},\{b\}\}$. If $f:(X,\tau,I)\to(Y,\sigma)$ is a function such that $f(A)=\{a\}$ and $f(X-A)=\{b\}$, then f is a non constant contra Λ_I^s-continuous function and (Y,σ) is a T_0-space, which contradicts the hypothesis. Therefore, (X,τ,I) is a Λ_I^s-connected space. □

Theorem 4.4. *If $f:(X,\tau,I)\longrightarrow(Y,\sigma)$ is a contra Λ_I^s-continuous function and (Y,σ) is a regular space, then f is Λ_I^s-continuous.*

Proof. Let $x\in X$ and U be an open subset of Y such that $f(x)\in U$. Since (Y,σ) is regular, there exists an open subset V of Y such that $f(x)\in V\subset Cl(V)\subset U$. Now, as f is contra Λ_I^s-continuous and $Cl(V)$ is a closed subset of Y containing $f(x)$, then by Theorem 3.3, there exists a Λ_I^s-open subset W of X such that $x\in W$ and $f(W)\subset Cl(V)\subset U$. By Theorem 3.1, we obtain that f is a Λ_I^s-continuous function. □

Definition 4.1. An ideal space (X,τ,I) is said to be Λ_I^s-*normal*, if for each pair of disjoint closed subsets A and B of X, there exist two disjoint Λ_I^s-open subsets U and V of X such that $A\subset U$ and $B\subset V$.

Remark 4.1. If (X,τ) is a normal space, then (X,τ,I) is a Λ_I^s-normal space for each ideal I on X.

Recall that a topological space (X,τ) is called *ultra normal* [8], if for each pair of nonempty disjoint closed subsets A and B of X, there exist two disjoint clopen subsets U and V of X such that $A\subset U$ and $B\subset V$. The following result shows that, the inverse image of an ultra normal space under a closed injective contra Λ_I^s-continuous function is a Λ_I^s-normal space.

Theorem 4.5. *If $f:(X,\tau,I)\to(Y,\sigma)$ is a closed injective contra Λ_I^s-continuous function and (Y,σ) is an ultra normal space, then (X,τ,I) is a Λ_I^s-normal space.*

Proof. Let A and B be two disjoint closed subsets of X. Since f is closed injective, $f(A)$ and $f(B)$ are disjoint closed subsets of Y and, as (Y,σ) is an ultra normal space, there exist two disjoint clopen subsets U and V of Y such that $f(A)\subset U$ and $f(B)\subset V$. Now, because f is contra Λ_I^s-continuous, it follows that $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint Λ_I^s-closed subsets of X such that $A\subset f^{-1}(U)$ and $B\subset f^{-1}(V)$. Therefore, (X,τ,I) is a Λ_I^s-normal space. □

Definition 4.2. An ideal space (X,τ,I) is said to be Λ_I^s-T_2, if for each pair of distinct points $x,y\in X$, there exist two disjoint Λ_I^s-open subsets U and V of X such that $x\in U$ and $y\in V$.

Remark 4.2. If (X,τ) is a T_2-space, then (X,τ,I) is a Λ_I^s-T_2-space for each ideal I on X.

Recall that a topological space (X,τ) is called *Urysohn* [9], if for each pair of distinct points $x,y\in X$, there exist two open subsets U and V of X such that $x\in U$, $y\in V$ and $Cl(U)\cap Cl(V)=\emptyset$. The following result shows that, the inverse image of a Urysohn space under an injective contra Λ_I^s-continuous function is a Λ_I^s-T_2-space.

Theorem 4.6. *If $f:(X,\tau,I)\to(Y,\sigma)$ is an injective contra Λ_I^s-continuous function and (Y,σ) is an Urysohn space, then (X,τ,I) is a Λ_I^s-T_2-space.*

Proof. Let x and y be two distinct points of X. Since f is injective, we have $f(x)\neq f(y)$ and, as (Y,σ) is an Urysohn space, there exist two open subsets U and V of Y such that $f(x)\in U$, $f(y)\in V$ and $Cl(U)\cap Cl(V)=\emptyset$. By Theorem 3.3, there exist two Λ_I^s-open subsets A and B of X such that $x\in A$, $y\in B$, $f(A)\subset Cl(U)$ and $f(B)\subset Cl(V)$. Thus, $f(A)\cap f(B)\subset Cl(U)\cap Cl(V)=\emptyset$, which implies that $f(A\cap B)=\emptyset$ and hence, $A\cap B=\emptyset$. This shows that (X,τ,I) is a Λ_I^s-T_2-space. □

According to [2], we say that a topological space (X,τ) is *locally indiscrete*, if each open subset of X is closed. In the following definition some modifications of a locally indiscrete space are introduced in order to investigate related properties with the functions defined in Section 3.

Definition 4.3. An ideal space (X,τ,I) is said to be:

(1) *Locally $\tau^\star$-indiscrete*, if each $\tau^\star$-open subset of X is closed in X.

(2) *Locally Λ_I^s-indiscrete*, if each Λ_I^s-open subset of X is closed in X.

(3) *Λ_I^s-space*, if each Λ_I^s-open subset of X is open in X.

Proposition 4.1. *Let (X,τ,I) be an ideal space. The following statements hold:*

(1) *If (X,τ,I) is locally Λ_I^s-indiscrete, then it is locally $\tau^\star$-indiscrete.*

(2) *If (X,τ,I) is locally $\tau^\star$-indiscrete, then it is locally indiscrete.*

(3) *(X,τ,I) is locally $\tau^\star$-indiscrete if and only if each $\tau^\star$-closed subset of X is open in X.*

(4) *(X,τ,I) is locally Λ_I^s-indiscrete if and only if each Λ_I^s-closed subset of X is open in X.*

(5) *(X,τ,I) is Λ_I^s-space if and only if each Λ_I^s-closed subset of X is closed in X.*

Theorem 4.7. *Let $f:(X,\tau,I)\to(Y,\sigma)$ be a contra Λ_I^s-continuous function. The following statements hold:*

(1) *If (X,τ,I) is locally Λ_I^s-indiscrete, then f is continuous.*

(2) *If (X,τ,I) is a Λ_I^s-space, then f is contra-continuous.*

Proof. (1) Let B be a closed subset of Y. Since f is contra Λ_I^s-continuous, $f^{-1}(B)$ is a Λ_I^s-open subset of X and, as (X,τ,I) is locally Λ_I^s-indiscrete, we obtain that $f^{-1}(B)$ is a closed subset of X. Therefore, f is a continuous function.

The proof of (2) is similar to that of (1). □

The following result shows that, the direct image of a Λ_I^s-space under a closed surjective contra Λ_I^s-irresolute (resp. contra quasi-Λ_I^s-continuous, contra Λ_I^s-continuous) function is a locally Λ_J^s-indiscrete (resp. locally $\sigma^\star$-indiscrete, locally indiscrete) space.

Theorem 4.8. *Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be a closed surjective function. The following statements hold:*

(1) *If f is contra Λ_I^s-irresolute and (X,τ,I) is a Λ_I^s-space, then (Y,σ,J) is locally Λ_J^s-indiscrete.*

(2) *If f is contra quasi-Λ_I^s-continuous and (X,τ,I) is a Λ_I^s-space, then (Y,σ,J) is locally $\sigma^\star$-indiscrete.*

(3) *If f is contra Λ_I^s-continuous and (X,τ,I) is a Λ_I^s-space, then (Y,σ) is locally indiscrete space.*

Proof. Let V be a Λ_J^s-open subset of Y. Since f is contra Λ_I^s-irresolute, $f^{-1}(V)$ is a Λ_I^s-closed subset of X and, as (X,τ,I) is a Λ_I^s-space, $f^{-1}(V)$ is a closed subset of X. Now, because f is closed surjective, $V=f(f^{-1}(V))$ is a closed subset of Y and hence, (Y,σ,J) is a locally Λ_J^s-indiscrete space.

The proofs of (2) and (3) are similar to that of (1). □

Recall that a topological space (X,τ) is said to be *strongly S-closed* [1], if each closed cover of X has a finite subcover. Now we introduce a modification of a strongly S-closed space using Λ_I^s-closed sets.

Definition 4.4. An ideal space (X,τ,I) is said to be *strongly S-Λ_I^s-closed*, if each cover of X by Λ_I^s-closed sets has a finite subcover.

Remark 4.3. If (X,τ,I) is a strongly S-Λ_I^s-closed space, then (X,τ) is a strongly S-closed space.

According to [7], we say that an ideal space (X,τ,I) is *Λ_I^s-compact* (resp. *$\tau^\star$-compact*), if each cover of X by Λ_I^s-open (resp. $\tau^\star$-open) sets has a finite subcover. The following result shows that, the direct image of a strongly S-Λ_I^s -closed space under a surjective contra Λ_I^s-irresolute (resp. contra quasi-Λ_I^s-continuous, contra Λ_I^s-continuous) function, is a Λ_J^s-compact (resp. $\sigma^\star$-compact, compact) space.

Theorem 4.9. *Let $f:(X,\tau,I)\to(Y,\sigma,I)$ be a surjective function. The following statements hold:*

(1) *If f is contra Λ_I^s-irresolute and (X,τ,I) is strongly S-Λ_I^s-closed, then (Y,σ,J) is Λ_J^s-compact.*

(2) *If f is contra quasi-Λ_I^s-continuous and (X,τ,I) is strongly S-Λ_I^s-closed, then (Y,σ,J) is $\sigma^\star$-compact.*

(3) *If f is contra Λ_I^s-continuous and (X,τ,I) is strongly S-Λ_I^s-closed, then (Y,σ) is compact.*

Proof. (1) Let $\{W_\alpha:\alpha\in\Delta\}$ be a cover of Y by Λ_J^s-open sets. Since f is contra Λ_I^s-irresolute, $\{f^{-1}(W_\alpha):\alpha\in\Delta\}$ is a cover of X by Λ_I^s-closed sets and, as (X,τ,I) is strongly S-Λ_I^s-closed, there exists a finite subfamily $\{f^{-1}(W_{\alpha_i}):i=1,\dots,n\}$ of $\{f^{-1}(W_\alpha):\alpha\in\Delta\}$ such that $X=\bigcup_{i=1}^{n} f^{-1}(W_{\alpha_i})$. Therefore $Y=f(X)=f\left(\bigcup_{i=1}^{n} f^{-1}(W_{\alpha_i})\right)=\bigcup_{i=1}^{n} f(f^{-1}(W_{\alpha_i}))=\bigcup_{i=1}^{n} W_{\alpha_i}$ and so, (Y,σ,J) is Λ_J^s-compact.

The proofs of (2) and (3) are similar to that of (1). □

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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